Math 70
Week 1

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1 Introduction. Events as Sets

What is *Probability*: Probability is a method to quantify the possibility of occurrence of events. Let's start with an example: assume we have a class of 50 students of boys and girls, some have green eyes and the others have blue eyes.

Class		
	Boys	Girls
Blue	5	10
Green	15	20

The teacher chooses one student uniform at random in the class. Say that student is S. Let's define two events $A = \{S \text{ is a boy}\}$, $B = \{S \text{ has green eyes}\}$. The sets A and B can be represented as yellow and red colored areas

	Boys	Girls
Blue	5	10
Green	15	20
A		

	Boys	Girls
Blue	5	10
Green	15	20
B		

S is chosen uniformly random, meaning that each student in the class have the same chance to be chosen, thus

$$P(\text{S is a boy}) = p(A) = \frac{\text{Number of Boys in the Class}}{\text{Total Number of Students}} = \frac{20}{50}$$

similarly

$$p(B) = \frac{\text{Number of Green Eyed Students in the Class}}{\text{Total Number of Students}} = \frac{35}{50}$$

$$p(A \cap B) = \frac{\text{Number of Green Eyed Boys in the Class}}{\text{Total Number of Students}} = \frac{15}{50}$$

$$p(A \cup B) = \frac{\text{Number of students either are boys or have Green Eyes in the Class}}{\text{Total Number of Students}} = \frac{40}{50}$$

Note that in this example, we defined the experiment (teacher choosing one student at random), and some the event (chosen student being a male or having green eyes), then question what is the possibility/probability of these particular events are happening.

Example 1.1. Two different numbers are chosen from the set $\{1, 2, 3, \ldots 50\}$. Find the probability that one is twice of the other.

Solution

Example 1.2. A casino offers the following game. They give you a four sided fair dice. First you roll the first dice, then they pay you the number of dollars that you roll. If the cost of this game is 3$, would you play it? What is your expected loss/profit? What if the cost was 2$. Would you change your answer?

Solution

Example 1.3. A casino offers the following game. They give you a four sided nonfair dice, so that probability of each number coming up is proportional to the number itself. First you roll the first dice, then they pay you the number of dollars that you roll. If the cost of this game is 3$, would you play it? What is your expected loss/profit?

Solution

Example 1.4. A casino offers the following game. They give you 2 four sided fair dice. First you roll the first dice, say that outcome is D_1. Then you roll the second dice D_1 times, and add up the all outcomes. Say this is P. Then they pay you P dollars. For example, you roll 3 in the first dice, so that $D_1 = 3$. Then you roll the second dice 3 times, say the outcomes are $4, 2, 4$. Then they pay you $P = 4 + 2 + 4 = 10\$$. If the cost of this game is 6\$, would you play it? What is your expected loss/profit?

Solution

Example 1.5. You are given an unfair coin, so that $P(Head) = \frac{1}{3}$. You throw your coin until you hit tails.

1. Compute the probability that you only roll twice.
2. Compute the probability that you roll at most four times.
3. Compute the probability that you roll at most seven times.

Solution

Example 1.6. You are given an unfair coin, so that $P(Head) = p \in (0, 1)$ is unknown. Your goal is to approximate p. How can you do it?

Solution

Example 1.7. At UCLA 70 percent of students can speak either French or Spanish respectively. If half of the UCLA students can speak Spanish and 30 percent of UCLA students can speak French, what percentage of students can speak both.

Solution

Example 1.8. (PSI 1.1.1) Of a group of patients having injuries, 28% visit both a physical therapist and a chiropractor and 8% visit neither. Say that the probability of visiting a physical therapist exceeds the probability of visiting a chiropractor by 16%. What is the probability of a randomly selected person from this group visiting a physical therapist?

Solution

Example 1.9. Westwood High has 300 students, and it involves three active clubs: Soccer club, Basketball club and Volleyball club. Except 16 students, everybody else is engaged in at least one of the clubs. There are 130 students enrolled in Soccer Club, 100 students enrolled in Basketball club and 144 students enrolled in Volleyball club. There are 30 students who plays both soccer and basketball, 40 students who play both soccer and volleyball and 32 students who play both basketball and volleyball. Find the number of students who play all.